

*Screening as a Unified Theory of Delinquency, Renegotiation, and Bankruptcy**

Natalia Kovrijnykh[†] and Igor Livshits[‡]

May 2013

Abstract

We propose a parsimonious model with adverse selection where delinquency, renegotiation, and bankruptcy all occur in equilibrium as a result of a simple screening mechanism. A borrower has private information about her cost of bankruptcy, and a lender may use random contracts to screen different types of borrowers. In equilibrium, some borrowers choose not to repay, and thus become delinquent. The lender renegotiates with some delinquent borrowers. In the absence of renegotiation, delinquency leads to bankruptcy. Presence of competition may induce the incumbent lender to renegotiate even when he would not do so in the monopoly setting. We apply the model to analyze effects of a government intervention in debt restructuring. We show that a mortgage modification program aimed at limiting foreclosures that fails to take into account private debt restructuring may have the opposite effect from the one intended.

Keywords: Default, Delinquency, Bankruptcy, Renegotiation, Adverse Selection, Screening, Consumer Credit

JEL Codes: D14, D82, D86, G18, G21

*We thank Andrei Kovrijnykh for getting us started on this project. We have also benefited from discussions with Hector Chade, Alejandro Manelli, Salvador Navarro, Andrei Savochkin, Jacob Short, and Galina Vereshchagina, and from comments by seminar participants at Arizona State University, Collegio Carlo Alberto, McMaster University, Ryerson University, and University of Iowa. Part of the work for this paper was carried out when Livshits was visiting Collegio Carlo Alberto; Livshits thanks the Collegio for their kind hospitality.

[†]Department of Economics, Arizona State University. Email: natalia.kovrijnykh@asu.edu.

[‡]Department of Economics, University of Western Ontario, and BERO. Email: livshits@uwo.ca.

1 Introduction

Default in consumer credit markets is not a simple binary event, but rather has multiple stages and possible outcomes. The first “stage” is delinquency, which is defined as being overdue on loan payments for a specified period of time (usually at least 60 days). Some, but not all, delinquent borrowers end up in bankruptcy. Lenders sometimes renegotiate with delinquent borrowers to prevent bankruptcy and achieve debt settlement.

We propose a very simple model where a single key friction generates all three phenomena — delinquency, renegotiation, and bankruptcy — as parts of an optimal arrangement. The friction is adverse selection — a borrower has private information about her cost of bankruptcy. We assume that the borrower is indebted to a single lender. To keep the model as simple as possible, we abstract from how the debt was acquired.¹ The lender offers repayment options to the borrower, and seeks to maximize the expected repayment. The alternative for the borrower to making the repayment is to file for bankruptcy. We focus on the case where the borrower’s cost of bankruptcy, unknown to the lender, can take one of two values, high or low.

Faced with adverse selection, the lender has two basic options when restricted to offering deterministic contracts. First, by asking for repayment that does not exceed the low-cost borrower’s willingness to pay, the lender can guarantee repayment from both types. Second, by asking for a greater repayment, the lender can extract more from the high-cost borrowers, but loses the low-cost type to bankruptcy.

The lender may be able to do better if he extracts different repayments from different types of borrowers. However, he cannot separate the two types of borrowers by offering a menu of deterministic contracts. The reason is that both types of borrowers have the same utility if they make the same repayment, so naturally every borrower will choose a lower repayment. But different types do have different utilities if they do *not* repay and end up in bankruptcy. The lender can utilize this feature and separate the two types of borrowers by using lotteries over repayments and bankruptcy. That is, the separation is possible because the two types of borrowers value lotteries in a different way, as their cost of bankruptcy is different.

We show that the optimal separating mechanism involves the lender offering a menu of random contracts that consists of a deterministic repayment and a lottery, aimed at

¹Endogenously determined debt can be easily incorporated into the model, as we show in the Appendix. Focusing on a single-period setup with exogenous debt highlights the simplicity of our mechanism and allows us to illustrate our results in the most parsimonious model possible.

the high- and low-cost borrowers, respectively. The lottery for the low-cost type is over a repayment that is lower than the deterministic one, and a very high repayment that exceeds the willingness to pay of both types. In this optimal mechanism, the high-cost borrowers make a higher repayment, while the low-cost borrowers decline that repayment and are then offered a better deal with some probability, but forced into bankruptcy with the complementary probability.

One of the central points of the paper is that such a mechanism has a natural economic interpretation and delivers the three phenomena — delinquency, renegotiation, and bankruptcy — described above. Indeed, offering the aforementioned menu is equivalent to making the following sequential offers. First, the lender offers a high repayment, which only high-cost borrowers accept. We interpret the borrowers who have agreed to make the high payment as having repaid the loan, while the borrowers who refuse to make it as becoming *delinquent*. Next, the lender offers a lower repayment to a fraction of the delinquent borrowers. We interpret the event of offering the lower repayment as *renegotiation*. The delinquent borrowers with whom the lender does not renegotiate declare *bankruptcy*.²

Renegotiation allows the lender to extract some repayment from the low-cost borrowers who reject the high repayment. However, the possibility of renegotiation makes delinquency more attractive and thus limits the amount that can be extracted from the high-cost borrowers. It is for this reason that the lender does not renegotiate with all delinquent borrowers. Thus, our paper also addresses the question of why we see some renegotiation in the consumer credit market but not all bankruptcies are avoided.

Having analyzed the interaction between the borrower and the monopolistic lender, we turn to studying an environment with competing lenders. The debt is owed to one lender — the incumbent, — and both the incumbent and outsiders offer contracts to the borrower. We assume that old debt is senior, so that if the borrower accepts an outsider's contract, the outsider has to repay the debt to the incumbent.

We show that the outsiders never renegotiate with the borrower, although the incumbent lender might. Moreover, the incumbent who did not renegotiate in the monopoly setting may choose to do so under competition. The reason is that competition limits the ability of the incumbent lender to extract repayment from borrowers. When repayment that can be extracted from the high-cost borrowers is sufficiently limited, the lender can renegotiate

²An implicit assumption needed for the sequential offers to be equivalent to the (simultaneous) menu offer is that the lender can commit not to renegotiate with all delinquent borrowers, for otherwise the high-cost borrowers will never agree to make the initial high repayment.

with the low-cost borrowers without distorting the decision of the high-cost ones. Hence, the probability of renegotiation under competition is (weakly) higher than that under monopoly.

We also illustrate that our model generates reasonable comparative statics predictions. In particular, we show that the bankruptcy rate is increasing in the debt level and is decreasing in the borrower's income.

Our model puts us in the unique position to analyze effects of a government intervention in consumer debt restructuring. One example of such an intervention is a mortgage modification program aimed at limiting foreclosures. Indeed, for an individual borrower, such an intervention is triggered by *delinquency*, offers debt restructuring — i.e., involves a *renegotiation*, — with the goal of avoiding *bankruptcy*, or foreclosure. Not only does our model capture all these stages of default, but, most importantly, it allows us to explicitly analyze the response of private lenders to the government intervention. We show that a government program that fails to take into account private debt restructuring may have the opposite effect from the one intended — rather than limiting the number of foreclosures, it may actually increase it. We also demonstrate how a seemingly irrelevant intervention can successfully prevent *all* defaults. Our analysis therefore illustrates that it is crucial for a policy maker designing such a program to take into account how private debt restructuring works, or else the program may backfire.

The rest of the paper is organized as follows. The next subsection reviews the related literature. Section 2 sets up the model with the monopolistic lender, and Section 3 characterizes the optimal contract in this model. Section 4 studies the environment with competing lenders. In Section 5, we present the comparative statics results. Finally, Section 6 analyzes the effects of a government intervention. Section 7 concludes.

1.1 Related Literature

Theoretical analysis of default in consumer credit markets has largely focused on bankruptcy and abstracted from delinquency, and especially renegotiation — see, for example, Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Livshits, MacGee, and Tertilt (2007), and many others. Notable exceptions are the papers by Chatterjee (2010), Adelino, Gerardi, and Willen (2013), Benjamin and Mateos-Planas (2012) and Athreya, Sanchez, Tam, and Young (2012). While Chatterjee (2010) makes a distinction between delinquency and

bankruptcy, he does not allow for renegotiation.³ Adelino, Gerardi, and Willen (2013), on the other hand, study renegotiation, but treat delinquency as exogenous. They document that renegotiations of delinquent mortgages are infrequent.⁴ In explaining this phenomenon, the authors point out that mortgage restructuring may not be ex-post profitable for the lenders as it foregoes to possibility of “self-cures” — delinquent mortgages being repaid in full. In contrast, in our model renegotiation is always profitable ex-post (i.e., after the borrower becomes delinquent, but generates an ex-ante distortion by affecting the incentive of high-cost borrowers to make the high repayment rather than choose delinquency. Thus, we view our explanation for why lenders do not renegotiate more frequently as complementary to that offered by Adelino, Gerardi, and Willen (2013).

Benjamin and Mateos-Planas (2012) and Athreya, Sanchez, Tam, and Young (2012) propose quantitative models with symmetric information and incomplete markets where all three stages of default are present. However, the mechanics of their models are very different from ours. In Benjamin and Mateos-Planas (2012), renegotiation occurs with an exogenously given probability, but the possibility of renegotiation leads to an endogenous distinction between delinquency and bankruptcy. In Athreya, Sanchez, Tam, and Young (2012), delinquency also triggers debt restructuring, but deterministically so.⁵ In contrast, in our model, the probability of renegotiation following delinquency is determined endogenously, and, as will be clear from Section 6, the endogeneity of renegotiation is crucial for policy analysis.

Another related paper is Hopenhayn and Werning (2008), who study a dynamic lending model where, like in our paper, the borrower has private information about her outside option. The optimal contract in their framework also features default occurring in equilibrium with positive probability. However, their model does not distinguish between delinquency and default (which is akin to bankruptcy in our setup), and thus does not allow for the possibility of renegotiation.

Unlike in the consumer debt literature, analysis of renegotiation has played a central role in the sovereign debt literature — see the seminal work by Bulow and Rogoff (1989) and more recent contributions by Kovrijnykh and Szentes (2007), Benjamin and Wright

³The distinction between bankruptcy and “informal bankruptcy” is also present in Dawsey and Ausubel (2004) and Dawsey, Hynes, and Ausubel (2009), but the informal bankruptcy is thought of as a terminal state, much like bankruptcy, rather than as a transitional stage that delinquency captures.

⁴Agarwal, Amromin, Ben-David, Chomsisengphet, and Evanoff (2011) also point out that lenders restructure merely a small fraction of delinquent mortgages.

⁵Coexistence of bankruptcy and delinquency in Athreya, Sanchez, Tam, and Young (2012) arises from an exogenously imposed additional cost of delinquency, namely income garnishment.

(2009), Yue (2010), Arellano and Bai (2012), and others. Our work differs from this strand of literature in a number of ways. One distinction is that the key friction in our paper is private information about the bankruptcy cost, which arguably is more relevant in consumer debt than sovereign debt context. Also, unlike the sovereign default papers, our model allows us to study an “extensive margin” of renegotiation, as the fraction of borrowers with whom the lender renegotiates is determined endogenously. This in turn allows us to analyze the effect of an intervention operating along this extensive margin.

From the modeling standpoint, our paper is closely related to papers by Maskin and Riley (1984), Matthews (1983), and Miller, Piankov, and Zeckhauser (2005). Maskin and Riley (1984) study a problem of designing an auction that maximizes the expected revenue of a seller of an indivisible good facing risk-averse bidders with unknown preferences. They show that making buyers bear risk relaxes incentive constraints. In addition, they find that the probability of winning the auction (obtaining the good) and the amount paid in the case of winning increase with a buyer’s valuation. Our result is similar in that, in our screening contract, a low-cost borrower makes a lower repayment, and with a lower probability, than a high-cost borrower. Matthews (1983) studies a similar problem to the one analyzed by Maskin and Riley (1984), but also analyzes the case where there is an unlimited supply of indivisible units sold. This case is closer to our setup, where it is possible for the lender to obtain repayments (which is analogous to selling a good) from multiple borrowers. Matthews (1983) finds that the optimal selling scheme gives some buyers only a probability of obtaining the good. Finally, Miller, Piankov, and Zeckhauser (2005) also consider a similar setup as the other two papers, but have the seller making sequential price offers. They show that the optimal selling scheme involves the seller making an offer that, if rejected, is followed by a subsequent, more attractive offer, but only with some probability. This selling scheme is similar to the sequential interpretation of the optimal contract in our model.

There are, however, important differences between our setup and the ones considered in these papers. First, in our model, different types of borrowers have identical payoffs from repaying but different payoffs from not repaying (declaring bankruptcy). In contrast, in the papers described above, the buyers differ in their utilities from obtaining the good, but derive identical utilities from not getting it. Thus, it may be possible to screen the buyers using lotteries over payments while selling a unit to each buyer with probability one (for instance, if there is unlimited supply of units, as in Matthews, 1983). In contrast, in our setup screening must involve some borrowers exercising their outside option. Second, these

papers impose interim participation constraints, while in our paper the borrower can refuse to “participate” in the mechanism *ex post*, i.e., after the outcome of a lottery is realized.

Notably, our application of screening through randomization to the environment of consumer credit generates a novel, unified theory of delinquency, renegotiation, and bankruptcy. Furthermore, our setting allows us to study the effects of competition — pressure put on the incumbent by outsiders — which is specific to a lending environment.

Finally, our analysis of the government intervention in debt restructuring contributes to the literature on the effects of the most notable such intervention in recent years — the Home Affordable Mortgage Program (HAMP), aimed at restructuring troubled mortgages and preventing foreclosures, which has been in place in the U.S. since 2009. Agarwal, Amromin, Ben-David, Chomsisengphet, Piskorski, and Seru (2012) offer a comprehensive empirical analysis of the effects of this program. The authors highlight the importance of accounting for changes in private restructuring in evaluating the effects of the program. Our theoretical model allows us to explicitly analyze the private sector’s response to an intervention, and to illustrate that it can lead to unexpected, and possibly undesired, consequences. These insights are complementary to the existing studies pointing out possible shortcomings of HAMP. Most notably, Mulligan (2009, 2010) points out severe distortions imposed by the means-testing aspect of the program that induces an excessively high effective income tax rate. Specifically, since the restructured payments depend directly on the borrower’s income, HAMP creates a strong incentive for the borrower to earn less. We treat income of borrowers as exogenous, thus ignoring such distortions. Instead, we highlight the distortions imposed by such a government program on the private sector debt renegotiation.

2 The Environment

We begin by studying a simple one-period environment with one lender and one borrower.⁶ The borrower is risk averse, and derives utility from consumption according to the utility function $u(c)$. The function u is continuous, strictly increasing, strictly concave, and satisfies the Inada condition $\lim_{c \rightarrow 0} u'(c) = +\infty$. The borrower has endowment I , known to everyone. We assume that the borrower is sufficiently indebted to the lender so that the lender can demand arbitrarily large repayments from her. We abstract from where the debt comes from. While endogenous debt can be easily incorporated into the model

⁶We can alternatively assume that there are many borrowers.

as illustrated in the Appendix, focusing on the single-period setup with exogenous debt highlights the simplicity of the mechanism we propose.

As an alternative to making repayments to the lender, the borrower has an option of declaring bankruptcy. The borrower’s cost of bankruptcy can be low or high, $\theta \in \{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$. This cost is known to the borrower, but is unobservable to the lender. The prior belief of the lender that the bankruptcy cost is high is denoted by γ , where $0 \leq \gamma \leq 1$. Alternatively, γ can be interpreted as the fraction of high-cost borrowers. If the borrower declares bankruptcy, she receives utility $v(I, \theta)$, while the lender receives nothing. The function $v(I, \theta)$ is strictly increasing in I for each θ , and $v(I, \theta_H) < v(I, \theta_L)$. Moreover, $u(0) < v(I, \theta) < u(I)$ for all I and θ .

The lender is risk neutral and maximizes the expected repayment that he extracts from the borrower. We assume that the lender makes a take-it-or-leave-it offer to the borrower. An offer consists of a menu of contracts, where each contract — which can be deterministic or random — specifies how much the borrower should repay to the lender. A deterministic contract is simply an amount R that the borrower is asked to repay; a random contract is a lottery over repayments. The borrower chooses one contract from the offered menu or rejects all contracts. In the latter case (or if the borrower does not make the repayment specified in the contract he chose) she has to declare bankruptcy.

3 Optimal Contracts with the Monopolistic Lender

Before considering possible contracts that the lender can offer in equilibrium, it will be useful to define $R_j(I)$ — the largest amount that a borrower with income I and bankruptcy cost θ_j is willing to repay. This repayment solves

$$u(I - R_j(I)) = v(I, \theta_j), \tag{1}$$

$j \in \{L, H\}$. By construction, the “willingness to repay” of the low-cost borrowers is lower than that of the high-cost borrowers: $R_L \leq R_H$.

3.1 Deterministic Contracts

Suppose first that the lender is restricted to offering a single deterministic contract. Depending on the level of the demanded repayment, denoted by R , three situations may arise. If $R \leq R_L$, then both types of borrowers will accept the contract. If $R \in (R_L, R_H]$, then

only high-cost borrowers will accept the contract, while low-cost borrowers will prefer to declare bankruptcy. Finally, if $R > R_H$, no borrower will accept the contract. Therefore, to maximize the expected repayment, the lender will offer either $R = R_L$ or $R = R_H$. We will refer to the first alternative as “pooling”, as it attracts both types of borrowers, and to the second one as “exclusion”, as it excludes — i.e., forces into bankruptcy — the low-cost borrowers.

Which of the two contracts generates higher profits to the lender will depend on the parameters of the model, in particular, on the fraction of high-cost borrowers, γ , and the extent to which the bankruptcy cost parameters, θ_H and θ_L , are different from each other.

3.2 Random Contracts

Since a deterministic contract specifies only the repayment, it is impossible to offer a menu of deterministic contracts and have different types of borrowers accepting different contracts. However, the lender may be able to achieve this by offering a menu of random contracts, as we will demonstrate below. We will refer to this case as “screening”, as the lender uses lotteries to screen the borrowers based on their cost of bankruptcy. As we only have two types of borrowers, we can, without loss of generality, limit the analysis to just two random contracts.

It is straightforward to see that the expected repayment is maximized by offering the following pair of contracts. The first contract is deterministic with repayment, which we denote by R_S , that attracts only the high-cost borrowers. The second contract is a lottery that offers a lower repayment with probability p and an implausibly large repayment (anything above R_H) with probability $1 - p$.⁷ To maximize the lender’s expected profit, the lower repayment in the second contract must be set to R_L : it maximizes the repayment extracted from the low-cost borrowers, and also minimizes the attractiveness of this contract to the high-cost borrowers. We denote the lottery by (R_L, p) .

Note that the only reason for p to be set strictly below one is to keep the high-cost borrowers from accepting the contract meant for the low-cost borrowers: if p were equal to one, the high-cost borrowers would never make the higher repayment offered to them. Indeed, profit maximization requires the deterministic repayment R_S to be such that the

⁷Offering such an implausibly large repayment is equivalent to simply offering the borrower the bankruptcy option.

high-cost type is just indifferent between the two contracts. That is,

$$u(I - R_S) = pu(I - R_L) + (1 - p)v(I, \theta_H) = pu(I - R_L) + (1 - p)u(I - R_H), \quad (2)$$

where the second equality follows from (1). Clearly, R_S is lower than R_H as long as $p > 0$, as offering the lottery will prevent extracting the full surplus from the high-cost type. Also, R_S is higher than R_L as long as $p < 1$, for otherwise the high-cost borrower's incentive constraint is lax and the lender could increase expected repayment by increasing R_S .

The lender's problem is then simply to choose p to maximize the expected repayment,

$$\max_{p \in [0,1]} \gamma R_S(p) + (1 - \gamma)pR_L, \quad (3)$$

where $R_S(p)$ is given by (2). Notice that choosing $p = 1$ and $p = 0$ corresponds to the pooling and exclusion scenarios, respectively. Therefore, the lender's problem is fully captured by the maximization problem (3) subject to constraint (2). Strict concavity of the utility function immediately implies that this problem has a unique solution, which we denote by p^* . The corresponding repayment by the high-cost type, $R_S(p^*)$, is denoted by R_S^* . We summarize the above discussion in the following proposition.

Proposition 1 *The repayment scheme that maximizes the lender's profits is to offer a menu consisting of a deterministic repayment R_S^* and a lottery (R_L, p^*) , where p^* solves (3) subject to (2).*

3.3 Sequential Interpretation of the Optimal Contract

One of the central points of the paper is that the simple screening mechanism described in the previous subsection generates the three stages of default in consumer credit — delinquency, renegotiation, and bankruptcy. In this subsection, we use a sequential setting to illustrate this point.

Suppose that instead of offering the two contracts simultaneously, the lender offers them sequentially. Assume also that the lender can commit ahead of time to (not) making offers. To be exact, he can commit to the probability of not making the second offer before the first offer is made. It is easy to see that under this assumption, the setup with sequential offers is equivalent to our original setup with simultaneous offers, and that the lender's problem is still (3) subject to (2).

In the sequential setting, the optimal contract described in Proposition 1 has the following interpretation. First, the lender offers a higher repayment, which only the high-cost borrowers accept. We interpret the low-cost borrowers who refuse to make the specified repayment as *delinquent*. Next, the lender offers a lower repayment to — i.e., *renegotiates* with — delinquent borrowers, but only with some probability. The borrowers with whom the lender renegotiates reach debt settlement, while the rest declare *bankruptcy*.

Notice that the assumption of commitment is crucial here. Without it, the lender would want to renegotiate with all borrowers who refused to make the initial high repayment. Of course, anticipating this, no one would make the high repayment to begin with.

3.4 Screening and Risk Aversion

We have described three possible strategies that the lender may follow: pooling, exclusion, and screening. Given the focus of the paper, the screening scenario is the most interesting of the three. Then the question arises: does the lender ever use screening — i.e., chooses $p \in (0, 1)$ — in equilibrium?

Interestingly, if borrowers were risk neutral, lotteries (and hence screening) would never be utilized in equilibrium. To see this, notice that with a linear utility function, equation (2) reduces to $R_S = pR_L + (1 - p)R_H$, and the lender's problem becomes

$$\max_{p \in [0,1]} pR_L + (1 - p)\gamma R_H.$$

Notice that the profits in the objective function is simply a linear combination of the profits under pooling and exclusion. That is, screening is always dominated by either pooling or exclusion (strictly so, unless $R_L = \gamma R_H$). Thus, the lender does not benefit from using random contracts.

With risk-averse borrowers, however, there are parameter values for which screening gives the lender a strictly higher payoff than the pooling and exclusion alternatives. This happens, for example, when $R_L = \gamma R_H$. At that point, the lender is indifferent between pooling and exclusion, as well as any screening menu consisting of the lottery (p, R_L) and the deterministic offer $\bar{R}(p) = pR_L + (1 - p)R_H$. Note that the low-cost borrowers are not affected by the riskiness of the lottery, as both outcomes generate the same utility for them (equal to their value of bankruptcy). Note further that a risk-neutral high-cost borrower would have been indifferent between the lottery (p, R_L) and the deterministic offer $\bar{R}(p)$. A risk-averse high-cost borrower, however, strictly prefers the latter, and thus the lender

is able to extract a higher payment $R_S(p) > \bar{R}(p)$ from her. As a result, the expected repayment is maximized by choosing some interior $p \in (0, 1)$.

Of course, there are parameter values for which either pooling or exclusion would be the lender's optimal strategies. In particular, exclusion (pooling) is attractive when γ is high (low) enough.

4 Competition among Lenders

So far we considered an environment with a single monopolistic lender. Now we will assume that there is an incumbent lender and a competitive fringe of outside lenders (or, equivalently, just one outside lender), and the borrower owes an amount D to the incumbent. The lenders simultaneously offer menus of contracts to the borrower. The borrower either chooses a contract from one of these menus or refuses all of them. We assume that the old debt is senior, which means that if the borrower accepts an outsider's contract and does not declare bankruptcy, the outsider must pay D to the incumbent.⁸ Notice that the level of debt only plays a role in the presence of competition, because it affects the outsiders' payoffs and thus their behavior. But if there is only one lender, then the actual level of debt is irrelevant in our model.

The first result in the competitive setting is that the outsiders never offer lotteries in equilibrium. In other words, an outsider offers the same terms to all borrowers, and never renegotiates with borrowers who reject his offer.

Proposition 2 *The outsider lenders never offer random contracts in equilibrium. They always offer a deterministic repayment equal to the debt level D .*

Proof: Suppose to the contrary that an outsider offers R for the high-cost type and (R', p) to the low-cost type. If both contracts are accepted, the resulting profit to the outsider is $\gamma(R - D) + (1 - \gamma)p(R' - D)$. It must be the case that the outsider generates zero profits, for otherwise the incumbent (or another outsider) could offer slightly lower repayments and earn positive profits. An even stronger result holds: the outsider must generate zero profits on each contract he offers, that is, $R - D = R' - D = 0$. Indeed, if the higher repayment exceeded D , then the outsider could earn positive profits by offering only that repayment. On the other hand, if the lower repayment was less than D , he could increase profits by

⁸Our key results do not change if we assume instead that an outsider needs to repay D even if the borrower declares bankruptcy.

not offering that repayment. Since $R = R' = D$, outsiders offer a single repayment equal to D . \square

Even though the outsiders never renegotiate with the borrower, the incumbent lender might. The reason is that for the incumbent debt is sunk cost, and thus he can earn less than D on some borrowers. As we will demonstrate later in this section, this is indeed what happens if the incumbent renegotiates with the borrower in the presence of competition: he earns exactly D on the high-cost borrowers, and strictly less than D on each of the low-cost borrowers. Furthermore, as we show below, the presence of competition may induce the incumbent to renegotiate even when a monopolistic lender would not have (would have chosen exclusion).

To illustrate these results, we consider how the outsiders' offers affect contracts offered by the incumbent. Notice first that if $D \leq R_L$, then the outsiders' offers generate pooling, while if $D \in (R_L, R_H]$, they correspond to exclusion. If $D > R_H$, outsiders can never generate positive profits, and therefore their presence is irrelevant. But when D is low enough, outsiders make sufficiently attractive offers, which puts a restriction on what the incumbent can offer.

In light of Proposition 2, the incumbent's problem under competition can be easily obtained from the monopolist's problem (2)–(3) by simply imposing additional constraints that the offered repayments cannot exceed D . Specifically, let $R_L^C = \min\{R_L, D\}$. Then the incumbent's problem becomes

$$\max_{p \in [0,1], R_S^C} \gamma R_S^C + (1 - \gamma)pR_L^C, \quad (4)$$

$$\text{s.t.} \quad u(I - R_S^C) = pu(I - R_L^C) + (1 - p)u(I - R_H), \quad (5)$$

$$R_S^C \leq D. \quad (6)$$

Let p_C^* denote the incumbent's optimal choice of p in the competitive environment. Note that when constraint (6) binds, p_C^* is pinned down by equation (5):

$$p_C^* = \frac{u(I - D) - u(I - R_H)}{u(I - R_L^C) - u(I - R_H)}. \quad (7)$$

In particular, when $D \leq R_L$, the above problem simply delivers $R_S^C = D$ and $p_C^* = 1$, which is the same (pooling) contract as the one offered by the outsiders.

The following proposition describes the types of contracts offered by the incumbent

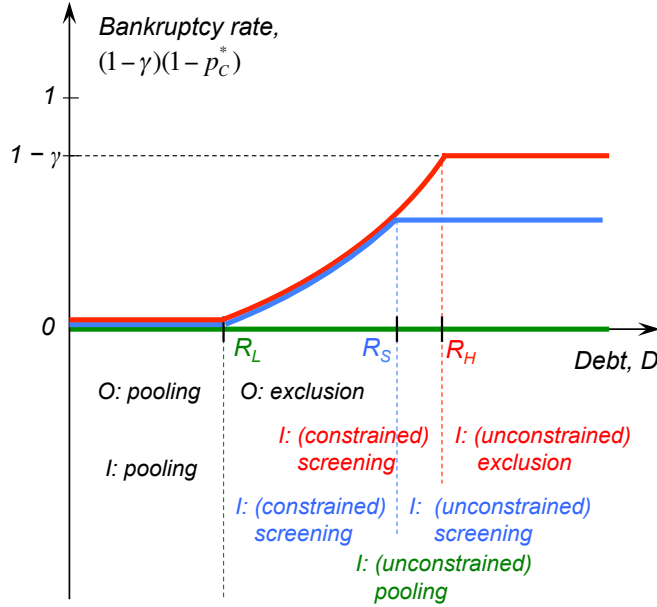


Figure 1: The probability of bankruptcy as a function of the debt level, and the corresponding types of equilibrium contracts. I and O stand for the incumbent and the outsiders, respectively.

depending on the level of debt and on what he would have offered in absence of competitors. These types of contracts are further illustrated on Figure 1, which plots the probability of bankruptcy $(1 - \gamma)(1 - p_C^*)$ as a function of the debt level D . In what follows, we will often refer to the probability of bankruptcy as the bankruptcy rate.

Proposition 3 (i) For $D \geq R_S^*$, the incumbent behaves as a monopolist.

(ii) For $D \leq R_L$, the incumbent and outsiders make the same pooling offers (where the repayment equals D).

(iii) If the incumbent chooses screening under monopoly, he also performs screening under competition for $D > R_L$.

(iv) If the incumbent chooses exclusion under monopoly, he switches to screening under competition for $D \in (R_L, R_H)$.

Part (i) follows immediately since R_S^* is the highest repayment offered by the monopolist. By Proposition 2, outsiders offer D . Therefore, when $D \geq R_S^*$, the outsiders' offers do not restrict the incumbent's choices, and thus he behaves as a monopolist.

When $D \leq R_L$, the offer of repayment equal to D is accepted by both types of borrowers. Clearly, the incumbent cannot offer a higher repayment, and would not find it profitable

to offer a lower one either. Thus in this case the incumbent and outsiders make the same pooling offers, as stated in part (ii) of the proposition, and all borrowers avoid bankruptcy.

If the incumbent uses screening under monopoly, he would be restricted to offer D to the high-cost type under competition when $D > R_L$. But this allows the incumbent to increase the probability of renegotiation to the point where the high-cost type again becomes indifferent between the deterministic repayment and the lottery. Therefore the incumbent will still perform screening under competition in this case, as shown in part (iii). The lower the level of debt, the higher the probability of renegotiation, and the lower the bankruptcy rate. (Since $D > R_L$, the probability of renegotiation will always remain strictly smaller than one.)

To understand part (iv), recall that under exclusion the monopolist extracts R_H from the high-cost borrowers. He does not find screening attractive when the expected repayment from the low-cost borrowers is not enough to offset the decrease in the repayment from the high-cost borrowers. But with competition, the incumbent can only extract $D(\in (R_L, R_H))$ from them anyway. Therefore he might as well offer R_L to the low-cost type, and pick the probability of renegotiation that makes the high-cost type just indifferent between the two offers.⁹

Proposition 3 contains an important result: presence of competition induces the incumbent to renegotiate more often (i.e., with a higher probability). In particular, the incumbent may start to renegotiate even when he would not do so in the monopoly setting. Intuitively, the only reason why the incumbent does not renegotiate with (all) delinquent borrowers is that the high-cost borrowers would then refuse to make the high repayment. By restricting this repayment, the presence of competition reduces the cost of renegotiation, causing the incumbent to renegotiate more often.

Introducing the face value of debt in this section allows us to think about debt forgiveness in addition to the three stages of default central to our analysis. Note that when competition is binding (which happens when $D \leq R_S^*$) constraint (6) holds with equality so that $R_S = D$, and thus high-cost borrowers fully repay their debt. But when the debt level is large enough ($D > R_S^*$), the incumbent asks for a repayment that is strictly below the debt level: $R_S < D$. Thus there is initial debt forgiveness for all borrowers. Low-cost borrowers refuse to make this payment, and we consider them delinquent. The lender renegotiates with a fraction of the delinquent borrowers, while the rest declare bankruptcy.

⁹Equation (2) provides a simple way to see this: if $R_S \in (R_L, R_H)$, then $p \in (0, 1)$.

5 Comparative Statics

In this section, we establish some key comparative statics results. Specifically, we focus on how the equilibrium bankruptcy rate varies with the borrower's income and debt.

5.1 Comparative Statics with Respect to Debt

As we have discussed in the previous section and illustrated on Figure 1, the bankruptcy rate is (weakly) increasing in the amount of debt. Therefore, the model generates reasonable comparative statics with respect to the debt level.

As can be seen from Figure 1, there are three “regions” of debt levels for a given income level. When debt is sufficiently low ($D < R_L$), it is always repaid in full, and there is no bankruptcy (or delinquency) in equilibrium. When the face value of debt is sufficiently high ($D > R_S^*$), competition is irrelevant, and thus so are marginal changes to the debt level. The incumbent behaves as a monopolist, and the bankruptcy rate is invariant to the debt level within this region. For intermediate levels of debt, the bankruptcy rate is strictly increasing in debt. We summarize the above results in the following claim.

Claim 1 *The equilibrium bankruptcy rate is increasing in the level of debt, strictly increasing for $D \in (R_L, R_S^*)$.*

5.2 Comparative Statics with Respect to Income

We now turn to the analysis of how the equilibrium contracts as well as the bankruptcy rate change with the level of the borrower's income. In order to derive analytical results, we turn to specific functional forms of the utility function and the value of bankruptcy. In particular, we restrict our attention to the CRRA utility function, $u(c) = c^{1-\sigma}/(1-\sigma)$, and the bankruptcy cost being a fraction of income, $v(I, \theta) = u((1-\theta)I)$.

We begin by establishing the following intermediate result:

Lemma 1 *Suppose that $u(c) = c^{1-\sigma}/(1-\sigma)$ and $v(I, \theta) = u((1-\theta)I)$. Then in the case with a monopolistic lender*

- (i) Repayments R_L , R_H , and R_S^* are proportional to the borrower's income;*
- (ii) The probability of bankruptcy, $(1-\gamma)(1-p^*)$, is independent of the borrower's income.*

Proof: With $v(I, \theta) = u((1 - \theta)I)$, equation (1) becomes $u(I - R_j) = u((1 - \theta_j)I)$, which immediately implies that $R_j = \theta_j I$ for $j \in \{L, H\}$. Furthermore, substituting $u(c) = c^{1-\sigma}/(1 - \sigma)$ into equation (2) and rearranging terms yields

$$\frac{(1 - R_S/I)^{1-\sigma}}{1 - \sigma} = p \frac{(1 - \theta_L)^{1-\sigma}}{1 - \sigma} + (1 - p) \frac{(1 - \theta_H)^{1-\sigma}}{1 - \sigma}.$$

Since the right-hand side of the above equation does not vary with I , the left-hand side does not either. Thus R_S is proportional to I for any p . Hence we can simply factor I out of the objective function (3), which implies that p^* does not depend on I . It then also follows that R_S^* is proportional to I . \square

Lemma 1 shows that if a monopolistic lender faces a borrower with higher income, he simply scales up the repayment(s) proportionally, but does not change the probability of renegotiation. Thus the bankruptcy rate is invariant to the borrower's income. This last result might sound undesirable at first glance as in reality high-income borrowers are presumably less likely to declare bankruptcy. Notice, however, that this result is established for “over-indebted” borrowers, i.e., borrowers whose debt exceeds R_S^* so that the incumbent behaves as a monopolist. And as part (i) indicates, this debt level is strictly increasing in the borrower's income. Since the bankruptcy rate is lower under competition than under monopoly, increasing income for a fixed level of debt eventually lowers the bankruptcy rate. Moreover, the bankruptcy rate under competition is itself decreasing in income, as Claim 2 below establishes.

Claim 2 *Suppose that $u(c) = c^{1-\sigma}/(1 - \sigma)$ and $v(I, \theta) = u((1 - \theta)I)$. Then in the presence of competition the probability of bankruptcy, $(1 - \gamma)(1 - p_C^*)$, is decreasing in the borrower's income I for any debt level D .*

Proof: If competition is non-binding ($D \geq R_S^*$) or sufficient to preclude bankruptcy altogether ($D \leq R_L$), then the statement holds trivially as the bankruptcy rate does not change with income — see Lemma 1. We want to establish that the bankruptcy rate decreases with income in the region of “constrained screening”, i.e., when $D \in (R_L, R_S^*)$. Since in that region constraint (6) is binding, constraint (7) (with $R_L^C = R_L$) becomes

$$p_C^* = \frac{u(I - D) - v(I, \theta_H)}{v(I, \theta_L) - v(I, \theta_H)}.$$

(Note that p_C^* is strictly decreasing in D , just as Proposition 1 suggests.) For $u(c) =$

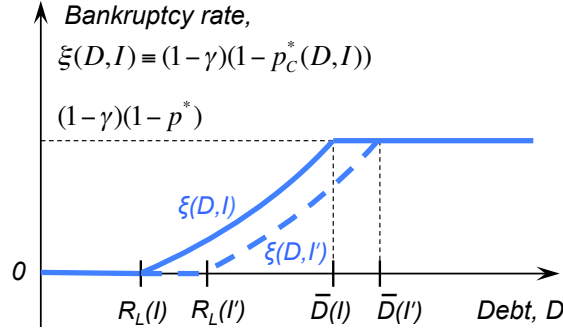


Figure 2: The probability of bankruptcy for two levels of income, I and I' , where $I' > I$.

$c^{1-\sigma}/(1-\sigma)$ and $v(I, \theta) = u((1-\theta)I)$, the above equation becomes

$$p_C^* = \begin{cases} [(1 - D/I)^{1-\sigma} - (1 - \theta_H)^{1-\sigma}] / [(1 - \theta_L)^{1-\sigma} - (1 - \theta_H)^{1-\sigma}], & \text{if } \sigma \neq 1, \\ [\ln(1 - D/I) - \ln(1 - \theta_H)] / [\ln(1 - \theta_L) - \ln(1 - \theta_H)], & \text{if } \sigma = 1. \end{cases}$$

The numerator in the first expression is strictly increasing (strictly decreasing) in I and the denominator is strictly positive (strictly negative) when $\sigma < 1$ ($\sigma > 1$). Thus the first expression is strictly increasing in I , and the same is true for the second expression. Hence for all σ , $(1 - \gamma)(1 - p_C^*)$ is strictly decreasing in I for a given level of debt D such that $D \in (R_L(I), R_S^*(I))$.¹⁰ Finally, the bounds of this interval are themselves strictly increasing in I (see Figure 2), as part (i) of Lemma 1 suggests. \square

The results of Lemma 1 and Claim 2 are illustrated on Figure 2.

To summarize, this section shows that our model generates reasonable comparative statics of the bankruptcy rate with respect to debt and income levels: a borrower with a lower income and/or higher debt is more likely to end up in bankruptcy.

6 Application: Government Intervention in Debt Restructuring

In this section, we use the framework that we have developed to analyze the effects of government intervention in debt restructuring. We show that understanding the workings of the private sector restructuring is crucial for designing a successful intervention.

¹⁰Since in the above expression p_C^* depends on I and D only through their ratio, this result also follows from Claim 1.

Consider, for instance, a government intervention in a form of a mortgage modification program that aims at lowering the foreclosure rate (which corresponds to the bankruptcy rate in our model).¹¹ One example of such a program is HAMP (Home Affordable Mortgage Program) introduced in the U.S. in 2009. We will analyze effects of a program of this sort through the lens of our model, and show that the program may have unintended consequences if its design is naive and ignores the effects on private debt restructuring.

Before we proceed, it is important to point out that in our model a government intervention is never Pareto improving (assuming that the government is subject to the same frictions as private lenders), because the equilibrium allocation is constrained Pareto efficient. In our analysis, we abstract from the reasons for the intervention, simply take it as exogenous, and focus on its effects.

Within our framework, we will assume that the government steps in if bankruptcy is initiated, that is, if private renegotiation has been unsuccessful (i.e., did not take place). To keep the analysis simple, we model the intervention as the government making an offer to a delinquent borrower with probability p_G to make a repayment R_G . If the borrower accepts the offer and makes the repayment, the repayment is transferred to the incumbent lender.

For simplicity, we restrict our attention to the case where the laissez-faire outcome is monopolistic screening. However, all of the results outlined below also hold in the presence of competition, assuming that the incumbent performs (constrained) screening. In analyzing the government intervention, we focus on the effect of the policy on the bankruptcy (foreclosure) rate.

6.1 Deterministic Intervention

We begin by characterizing the simplest case where the government intervention is deterministic, i.e., $p_G = 1$. We will illustrate most of our results in this simple case, and then show that some additional insights can be obtained in the case of random intervention.

Notice first that if the repayment R_G offered by the government exceeds R_H , then the intervention is completely irrelevant, because no borrower will ever want to make such a repayment. Thus, we can view the case of $R_G \geq R_H$ as the no-intervention benchmark.

Consider next what happens if $R_G \leq R_L$, i.e., if the government offers a repayment

¹¹The motivation for the government intervention may come from trying to limit the deadweight loss arising from foreclosures and/or out of concern for spill-overs through depressed house prices or “broken windows,” etc.

that is lower than the lender's offer to delinquent borrowers in absence of an intervention. Clearly, such an intervention constrains the lender because no borrower would accept a higher repayment knowing that she would be offered the more favorable R_G upon rejecting the lender's offer. Thus, the effect of the intervention in this case is similar to the effect of competition with $D = R_G \leq R_L$: a pooling outcome is achieved (i.e., all borrowers repay R_G) and the bankruptcy rate inevitably drops to zero. Thus, in this case, the government policy is (trivially) effective, as it prevents all bankruptcies in equilibrium.

Finally, consider the less trivial case of $R_G \in (R_L, R_H)$, where the repayment offered by the government exceeds the willingness to pay of the low-cost borrowers, but is acceptable to the high-cost borrowers. In this case, the government intervention only restricts the lender's ability to extract repayment from the high-cost borrowers.

Recall from Section 4 that when $D \in (R_L, R_S^*)$, the presence of competition forces the incumbent lender to renegotiate more often and thus reduces the bankruptcy rate. Since the incumbent's ability to extract repayment from the high-cost type is limited by competition anyway, he can extract repayment from a higher fraction of the low-cost type without distorting the incentives of the high-cost type. By analogy, one might infer that the government intervention with $R_G \in (R_L, R_H)$ would have a similar effect and reduce the bankruptcy rate. However, in what follows, we show that the restriction imposed on the lender by the government is in fact quite different from the one imposed by the competition. Moreover, in some cases, the bankruptcy rate actually increases in response to the intervention.

Formally, when $R_G \in (R_L, R_H)$ and $p_G = 1$, the lender's problem becomes

$$\max_{p \in [0,1]} \gamma \hat{R}_S(p) + (1 - \gamma)pR_L, \quad (8)$$

where $\hat{R}_S(p)$ is given by

$$u(I - \hat{R}_S) = pu(I - R_L) + (1 - p)u(I - R_G). \quad (9)$$

Note that the problem is identical to the familiar (3) subject to (2), where R_H has been replaced by R_G . That is, the government intervention basically amounts to lowering the high-cost borrowers' willingness to repay, R_H . Notice also that problem (8)–(9) is quite different from the problem of the incumbent lender under competition, (4)–(6). We denote the solution to problem (8)–(9) by \hat{p} .

Notice that in equilibrium no borrower actually makes the repayment offered by the government. The low-cost borrowers reject the government's offer because R_G exceeds their willingness to pay, and the high-cost borrowers never receive the offer in the first place, because the lender makes them an offer that they prefer to delinquency. Thus, all renegotiation is performed by the lender, and the equilibrium bankruptcy rate is $(1 - \gamma)(1 - \hat{p})$.

In order to understand the effects of the intervention, we will study comparative statics of \hat{p} with respect to R_G , keeping in mind that $R_G \geq R_H$ corresponds to the laissez-faire case. We will then compare the bankruptcy rate obtained under $R_G \in (R_L, R_H)$ with that under $R_G = R_H$.

To this end, consider the first order condition of the lender's problem (8)–(9). It can be written as

$$(1 - \gamma)R_L = \gamma \frac{u(I - R_L) - u(I - R_G)}{\underbrace{u'(I - \hat{R}_S(p; R_G))}_{= \frac{d\hat{R}_S}{dp}}}, \quad (10)$$

where $\hat{R}_S(p; R_G)$ is defined by (9). The left-hand side of the above equation is the marginal benefit of increasing p — it corresponds to an increase in the lender's profits due to a higher total repayment from the low-cost borrowers (and is unaffected by R_G). The right-hand side is the marginal cost of an increase in p — it reflects the fact that \hat{R}_S must be reduced as p increases to keep the incentive constraint 9 satisfied.

The rate at which \hat{R}_S can be “exchanged” for p , $d\hat{R}_S/dp$, depends on R_G through two channels. First, as R_G falls, the high-cost borrowers' utility from the lottery increases, and thus a smaller increase in utility $u(I - \hat{R}_S)$ is needed to keep (9) satisfied as p increases. This effect is reflected in the numerator of the right-hand side of (10) being increasing in R_G . The second effect, working in the opposite direction, comes from the fact that as R_G falls, so does \hat{R}_S , which lowers the marginal utility $u'(I - \hat{R}_S)$. This in turn increases the rate at which an increase in $u(I - \hat{R}_S)$ translates into a decrease in \hat{R}_S . This second effect is reflected in the denominator of the right-hand side of (10) being increasing in R_G .

Whether the marginal benefit of an increase in p , $\gamma d\hat{R}_S/dp$, increases or decreases with R_G depends on which of the two effects dominates. Suppose, for example, that R_H is very close to I , and R_G decreases from R_H marginally. Since bankruptcy is arbitrarily costly for the high-cost borrowers, even a small probability of bankruptcy is enough to make delinquency unattractive for them, and to induce them to make the prescribed payment.¹² This

¹²This follows from the assumption that the utility function satisfies the Inada condition.

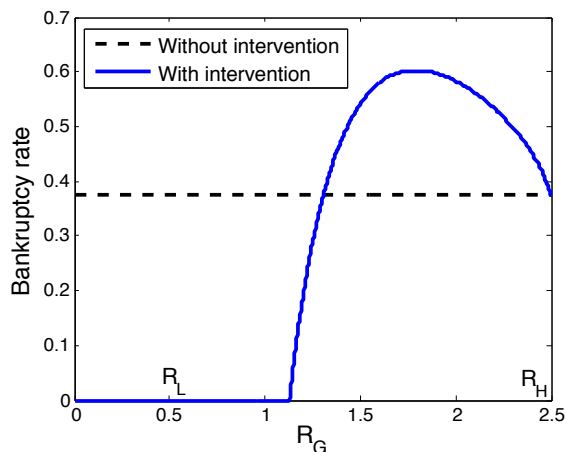


Figure 3: The bankruptcy (foreclosure) rate as a function of R_G , where $p_G = 1$. Parameter values: $I = 2.55$, $R_L = 0.5$, $R_H = 2.5$, $\gamma = 0.4$.

implies that $u'(I - \hat{R}_S(p; R_G))$ is very responsive to the change in R_G , so that the negative effect dominates and thus the probability of renegotiation decreases as R_G decreases. But as R_G falls close to R_L , the numerator on the right-hand side of (10) becomes small, and the benefit of increasing \hat{p} becomes greater than the cost. Thus, for R_G close enough to R_L , the positive effect dominates, and the intervention causes the lender to choose pooling as the optimal contract, i.e., $\hat{p} = 1$.

We have thus established that \hat{p} is generally non-monotone in R_G . Most interestingly, \hat{p} decreases, and thus the bankruptcy rate rises, in response to the government intervention if $R_G (< R_H)$ is close enough to I . That is, the government intervention aimed at preventing foreclosures may actually lead to an increase in foreclosures in equilibrium.

Figure 3 demonstrates the non-monotonicity of the bankruptcy rate as a function of R_G when $p_G = 1$ using in a numerical example. In these computations, we use the logarithmic utility function $u(c) = \ln c$ and assume that the cost of bankruptcy is proportional to income, i.e., $v(I, \theta) = u(I(1 - \theta))$.¹³ The borrower's income I is set to 2.55, and we pick the cost parameters θ_L and θ_H such that $R_L = 0.5$ and $R_H = 2.5$. The fraction of high-cost borrowers γ is set to 0.4. The solid and dashed lines on Figure 3 correspond to the bankruptcy rates with and without the government intervention, respectively. Since the government intervention is irrelevant when $R_G = R_H$, the two lines coincide at that point.

Two scenarios illustrated on Figure 3 are of particular interest. First, consider the case

¹³Note that with the log utility, this is equivalent to a “stigma” cost of bankruptcy, that is, a fixed additive utility cost: $v(I, \theta) = \ln(I) + \ln(1 - \theta)$.

where R_G is greater than R_L but is sufficiently close to it. In this case, the government intervention is completely successful in preventing foreclosures, despite appearing irrelevant — the repayment offered by the government is greater than that offered by the lender, as the lender offers R_L to all borrowers. That is, in equilibrium the government is not actively involved in restructuring mortgages. Yet, absent the intervention, the bankruptcy rate would have been strictly positive (see the point $R_G = R_H$).

Second, consider the case where R_G is below R_H but is sufficiently close to it. In this case, the government intervention “backfires” — it leads to an increase rather than a decrease in foreclosures. As we argued earlier, in this case the government offer is never accepted in equilibrium. In the next subsection we will demonstrate a case where the intervention backfires even though borrowers who receive the government offer, accept it.

6.2 Random Intervention

Next, we consider the case of a random intervention, $p_G < 1$, which can be interpreted as the borrower not being certain whether she is eligible for the government program. We will illustrate two additional scenarios that arise in this case that were absent in the case of the deterministic intervention.

In the first scenario, the policy is totally ineffective (i.e., it does not change the bankruptcy rate) although the government is busy preventing foreclosures. In the second scenario, the policy again backfires (leads to more foreclosures), but unlike in the case of the deterministic intervention, the government offer is accepted by some borrowers.

These scenarios are illustrated on Figure 4 using a numerical example similar to the one considered in the previous subsection. The probability of the government making an offer, p_G , is set to 0.3. Apart from the different p_G , the example depicted on the right panel of Figure 4 has all the same parameters as that on Figure 3. On the left panel, only the proportion of high-cost borrowers is different: $\gamma = 0.1$. Again, the point $R_G = R_H$ corresponds to no intervention.

First, consider the case where $R_G = R_L$ and $p_G \leq p^*$ (which is the case on both panels of Figure 4). Without the intervention, the lender sets p equal to p^* . Anticipating the intervention, the lender simply adjusts the probability of renegotiation to offset the intervention, i.e., $\hat{p} + (1 - \hat{p})p_G = p^*$. The resulting bankruptcy rate, $(1 - \gamma)(1 - p^*)$, is same as the laissez-faire one, and thus the intervention is ineffective. In this case, the government is busy preventing foreclosures, but its net effect is exactly nil.

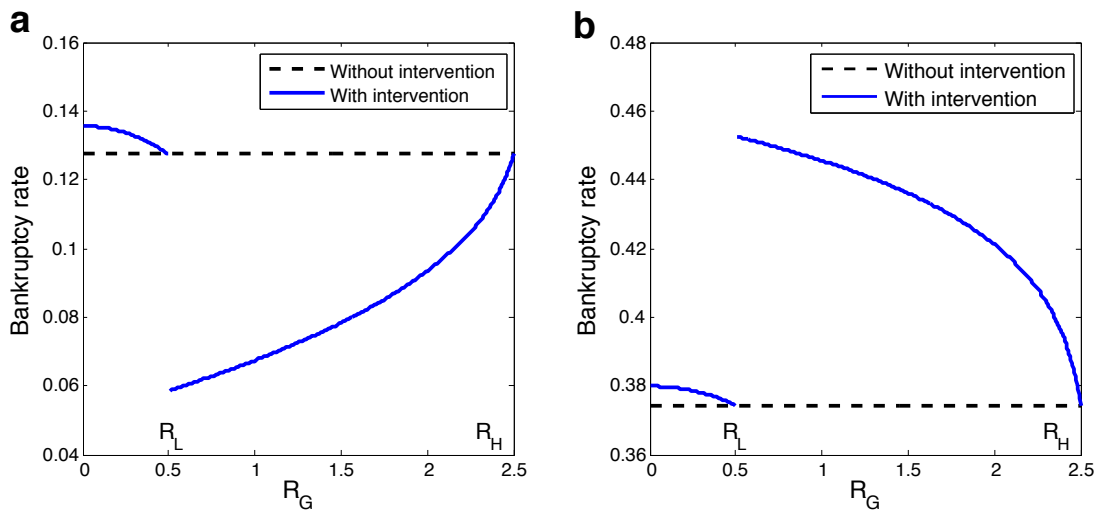


Figure 4: The bankruptcy (foreclosure) rate as a function of R_G , where $p_G = 0.3$. Parameter values: $I = 2.55$, $R_L = 0.5$, $R_H = 2.5$; panel a: $\gamma = 0.1$, panel b: $\gamma = 0.4$.

Next, consider the case where $R_G < R_L$. Recall from the previous section that when $p_G = 1$, such an intervention necessarily leads to pooling, i.e., reduces the bankruptcy rate to zero. This is not necessarily the case when $p_G < 1$. In fact, as Figure 4 shows, a random offer from the government with a repayment that is lower than that offered to delinquent borrowers by the lender ($R_G < R_L$) can lead to an increase in the number of foreclosures.¹⁴ In this case, the government program once again backfires. Compared to the case with deterministic intervention, now the government is actively participating in reducing foreclosures, as its offer (of a lower repayment) will be accepted in equilibrium by the delinquent borrowers. Yet, the foreclosure rate is higher than it would have been in the absence of the intervention.

Finally, when $R_G \in (R_L, R_H)$, the equilibrium bankruptcy rate can also be lower or higher than without intervention, as illustrated on the left and right panels of Figure 4, respectively. Recall that the model parameters for the two panels only differ in the proportion of high-cost borrowers, γ . When γ is higher, the lender has less incentives to renegotiate with delinquent (low-cost) borrowers, and so is more likely to behave adversely (i.e., renegotiate less often) in response to the intervention. Finally, notice that in the example on the right panel, an intervention with any offer R_G leads to the bankruptcy rate

¹⁴The argument behind this result is similar to the argument behind the non-monotonicity result described in the previous subsection, except now the government offer affects the lender's ability to extract repayment not only from the high-cost type, but also from the low-cost type.

that is at least as high as in the laissez-faire case.

The results presented in this section indicate that explicit modeling of the private sector debt restructuring is key for analyzing the effects of a government intervention. In particular, the failure to understand how private lenders renegotiate with delinquent borrowers can lead to the policy having the opposite effect from the one intended.

7 Conclusions

We propose a simple model of consumer credit where a lender demands repayments from an indebted borrower, and the borrower's alternative to making a repayment is to declare bankruptcy. The main friction in the model is that the borrower's cost of bankruptcy is her private information.

We characterize the optimal contract in this environment. We show that the lender may choose to screen different types of borrowers using lotteries over repayments. The optimal screening contract has a natural economic interpretation as it generates three stages of default — delinquency, bankruptcy, and renegotiation. Specifically, the lender first offers a high repayment that only borrowers with the high bankruptcy cost accept. Borrowers with the low bankruptcy cost refuse to make this payment, and are thus considered delinquent. The lender then renegotiates by offering a lower repayment, but only with a fraction of the delinquent borrowers, while the rest end up in bankruptcy.

We show that in a setting with competitive lenders where old debt is senior, outsiders never renegotiate with the borrower. However, the pressure from competitors may induce the incumbent lender to renegotiate even when he would not do so as a monopolist.

We apply the model to analyze the effects of a government intervention in debt restructuring, such as a mortgage modification program. We show that a program aiming to reduce foreclosures that overlooks the response of the private debt restructuring may lead to an increase rather than a reduction in the bankruptcy rate.

Appendix: A Two-Period Model of Debt

This section presents a simple two-period model that endogenizes the debt level acquired in the first period, and has our basic mechanism at work in the second period.

Consider an environment with one borrower and several identical lenders. There are two periods, $t = 1, 2$. Assume for simplicity that the borrower's endowment in period 1 is zero. Her endowment in period 2, as well as her cost of declaring bankruptcy in that period, are random and unknown to everybody in period 1. Once the uncertainty is realized in period 2, the realization of the borrower's endowment is public knowledge, while her cost of bankruptcy is her private information.

We assume that the markets are incomplete: period 1 contracts cannot be made contingent on the realization of endowment (nor on the cost of bankruptcy) in period 2. Contracting in period 1 is restricted to specifying a transfer of resources to the borrower, c_1 , and the face value of debt in period 2, D .¹⁵ Lenders compete in contracts (c_1, D) that they offer to the borrower, and the borrower picks one contract or rejects all contracts (the latter option means living in autarky). The lender whose contract is accepted becomes the incumbent. In period 2, once uncertainty is realized, the borrower, the incumbent lender, and the other lenders interact in the environment described in the main text given the debt level D and the endowment realization I .

The borrower's preferences are represented by

$$u(c_1) + \beta E[(1 - \chi)u(I - R(D, I)) + \chi v(I, \theta)],$$

where $R(D, I)$ is the repayment in period 2 and χ is the indicator of bankruptcy. The expectation in the above expression is then taken over the endowment in the second period, the bankruptcy cost, and, if applicable, any contractual randomness.

The lenders have deep pockets and maximize $-c_1 + E[R(D, I)]/(1 + r)$, where r denotes the risk-free interest rate. Competing lenders earn zero profits in period 1, and thus (c_1, D) maximizes the borrower's expected utility in the first period subject to the lenders' zero expected profit condition. The equilibrium level of D thus solves the following problem:

$$\max_D u \left(\frac{1}{1 + r} E[R(D, I)] \right) + \beta E[(1 - \chi)u(I - R(D, I)) + \chi v(I, \theta)].$$

¹⁵In other words, c_1 is the amount borrowed and D/c_1 is the gross interest rate. Note that the interest rate depends on the loan size.

Let D^* denote the solution to this problem.

In this model, the type of contract offered by the incumbent in the second period will vary depending on the income realization. Given the equilibrium level of debt D^* , there is a threshold \bar{I} such that $R_L(I) > D^*$ for all $I > \bar{I}$, so that the equilibrium in the corresponding states is competitive pooling, where all borrowers repay their debt. There is also a second threshold $\underline{I} \in (0, \bar{I}]$ such that $R_H(I) < D^*$ for all $I < \bar{I}$, so that in the corresponding income states, the incumbent behaves as a monopolist. In this case, the incumbent lender grants partial debt forgiveness to all borrowers, and then might renegotiate further with a fraction of low-cost borrowers who refuse to make the specified payment. For $I \in (\underline{I}, \bar{I})$, we have $D^* \in (R_L(I), R_S^*(I))$, which implies that the incumbent lender performs constrained screening. In this region, only the high-cost borrowers fully repay their debt, and the incumbent renegotiates with some low-cost borrowers. One can easily construct a distribution of period 2 incomes that results in all three cases occurring in equilibrium with positive probability. Thus, this extension illustrates that the mechanism highlighted in the paper applies if the level of debt is determined endogenously.

References

- ADELINO, M., K. GERARDI, AND P. WILLEN (2013): “Why Don’t Lenders Renegotiate More Home Mortgages? Redefaults, Self-Cures and Securitization,” mimeo.
- AGARWAL, S., G. AMROMIN, I. BEN-DAVID, S. CHOMSISENGPHET, AND D. D. EVANOFF (2011): “Market-Based Loss Mitigation Practices for Troubled Mortgages Following the Financial Crisis,” Federal Reserve Bank of Chicago Working Paper 2011-03.
- AGARWAL, S., G. AMROMIN, I. BEN-DAVID, S. CHOMSISENGPHET, T. PISKORSKI, AND A. SERU (2012): “Policy Intervention in Debt Renegotiation: Evidence from the Home Affordable Modification Program,” Dice Center Working Paper 2012-20.
- ARELLANO, C., AND Y. BAI (2012): “Linkages Across Sovereign Debt Markets,” mimeo.
- ATHREYA, K., J. M. SANCHEZ, X. S. TAM, AND E. R. YOUNG (2012): “Bankruptcy and Delinquency in a Model of Unsecured Debt,” Federal Reserve Bank of St. Louis Working Paper 2012-042B.
- BENJAMIN, D., AND X. MATEOS-PLANAS (2012): “Formal versus Informal Default in Consumer Credit,” mimeo.

- BENJAMIN, D., AND M. L. WRIGHT (2009): “Recovery Before Redemption: A Theory Of Delays In Sovereign Debt Renegotiations,” mimeo.
- BULOW, J., AND K. ROGOFF (1989): “A Constant Recontracting Model of Sovereign Debt,” *Journal of Political Economy*, 97(1), 155–178.
- CHATTERJEE, S. (2010): “An Equilibrium Model of the Timing of Bankruptcy Filings,” mimeo.
- CHATTERJEE, S., D. CORBAE, M. NAKAJIMA, AND J.-V. RÍOS-RULL (2007): “A Quantitative Theory of Unsecured Consumer Credit with Risk of Default,” *Econometrica*, 75(6), 1525–1589.
- DAWSEY, A. E., AND L. M. AUSUBEL (2004): “Informal Bankruptcy,” mimeo.
- DAWSEY, A. E., R. M. HYNES, AND L. M. AUSUBEL (2009): “The Regulation of Non-Judicial Debt Collection and the Consumers Choice among Repayment, Bankruptcy and Informal Bankruptcy,” John M. Olin Law and Economics Research Paper Series No. 2009-13.
- HOPENHAYN, H., AND I. WERNING (2008): “Equilibrium Default,” mimeo.
- KOVRIJNYKH, N., AND B. SZENTES (2007): “Equilibrium Default Cycles,” *Journal of Political Economy*, 115(3), 403–446.
- LIVSHITS, I., J. MACGEE, AND M. TERTILT (2007): “Consumer Bankruptcy: A Fresh Start,” *American Economic Review*, 97(1), 402–418.
- MASKIN, E., AND J. RILEY (1984): “Optimal Auctions with Risk Averse Buyers,” *Econometrica*, 52(6), 1473–1518.
- MATTHEWS, S. (1983): “Selling to Risk Averse Buyers with Unobservable Tastes,” *Journal of Economic Theory*, 30, 370–400.
- MILLER, N., N. PIANKOV, AND R. ZECKHAUSER (2005): “Possibly-Final Offers,” mimeo.
- MULLIGAN, C. B. (2009): “Means-Tested Mortgage Modification: Homes Saved or Income Destroyed?,” NBER Working Paper No. 15281.
- (2010): “Foreclosures, Enforcement, and Collections under the Federal Mortgage Modification Guidelines,” NBER Working Paper No. 15777.

YUE, V. Z. (2010): "Sovereign Default and Debt Renegotiation," *Journal of International Economics*, 80(2), 176–187.