

# Designing Optimal Pension Systems

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# Introduction

Labor supply decisions are affected by ..

- labor income taxes - primarily via intensive labor supply margin
- pension system - primarily via extensive labor supply margin

## Question

- What does optimal policy look like with both intensive and extensive margins of labor supply?

Approach:

- integrate extensive margin of labor supply into Mirrleesian framework

# Preview of results

We develop a theoretical framework that..

- provides a direct test of whether a pension system is Pareto optimal
  - a simple testable relationship between intensive and extensive labor distortions and labor supply elasticity that any Pareto optimal system must satisfy
- qualitatively justifies large disincentives to work after retirement
  - a jump in taxes after retirement age

Note: first statement is about optimal allocation, second about implementation

# Outline of the rest of the talk

- ① Full information benchmark
- ② Adding private information
- ③ Optimal Allocations
- ④ Implementation
- ⑤ A way forward

# Full Information Benchmark

# Setup

- Time:  $t \in [0, \infty)$
- At each date a generation is born with mass 1
- Each generation lives for 1 unit of time from  $t$  to  $t + 1$
- Each agent in generation  $t$  has a type  $j \in \{1, \dots, n\}$
- Measure  $\pi_j$  of agents of type  $j$

# Consumers

- $c_t(a, j)$  : consumption of agent born at  $t$ , age  $a$ , type  $j$
- $l_t(a, j)$  : hours worked

- Preferences:

$$\int_0^1 e^{-\rho a} [u(c_t(a, j)) - \tilde{v}(l_t(a, j))] da$$

- $u(\cdot)$ :  $C^2$ , st. inc and concave,  $u'(0) = \infty$
- Fixed cost of entering work force:

$$\tilde{v}(l) = v(l) + \eta \cdot \mathbf{1}_{[l > 0]}$$

- $v(\cdot)$ :  $C^2$ , st. inc and convex,  $v'(0) = 0$



# Technology

- Aggregate production function:

$$F(K, L) = (1 + r)K + L$$

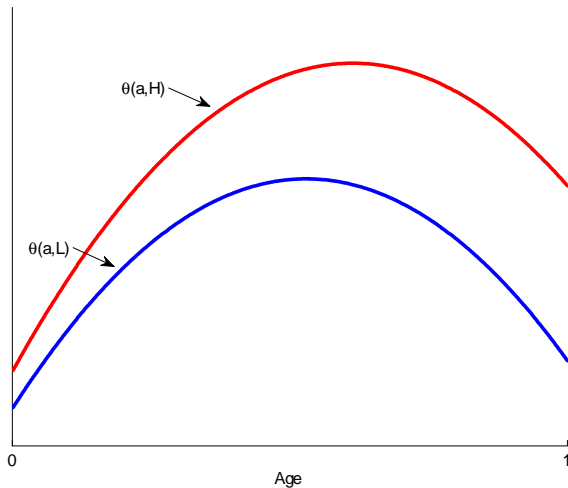
where  $L$  is total effective hours worked

$$L = \sum_{j=1}^n \pi_j \theta(a, j) l(a, j)$$

- $\theta(a, j)$  is productivity profile over life cycle

# Productivity profile over life cycle

$\theta(a, j)$  is inverse-U-shaped in age dimension



## A modification - taste shocks

- We focus on a version of the model with taste shocks,  $\phi$
- Preferences of a  $j$ -type agent:

$$\int_0^1 e^{-\rho a} \left[ u(c_t(a, j)) - \phi(a, j)v(y_t(a, j)) - \eta \mathbf{1}_{[y_t(a, j) > 0]} \right] da$$

- where  $y$  is output and  $\phi(a, j)$  is U-shaped along  $a$ -dimension
- If  $v(l) = l^{1+\gamma}/(1+\gamma)$ , then setting  $\phi(a, j) = \frac{1}{\theta(a, j)^{1+\gamma}}$  makes the two formulations equivalent

# Simplifying assumptions

- $\rho = 0, r = 0$ 
  - no growth in consumption
  - indifference in timing of labor supply
- Focus on stationary allocations  $\{c(a, j), l(a, j)\}$ 
  - since no aggregate uncertainty, no growth

# Adding Private Information

# Adding private information

- Type is unobservable
- Age, consumption, and output are observable
- If type  $j$  pretends to be  $i$ , he receives allocation  $\{c(a, i), y(a, i)\}$  for all  $a \in [0, 1]$
- Incentive constraints, for all  $j$

$$j \in \arg \max_i \int_0^1 [u(c(a, i)) - \phi(a, j)v(y(a, i)) - \eta \mathbf{1}_{[y(a, \theta) > 0]}] da$$

# Mechanism design problem

$$\max_{c(a,j), y(a,j)} \sum_j \pi_j \int_0^1 [u(c(a,j)) - \phi(a,j)v(y(a,j)) - \eta \mathbf{1}_{[y(a,j)>0]}] da$$

s.t.

$$\sum_j \pi_j \int_0^1 c(a,j) da + G \leq \sum_j \pi_j \int_0^1 y(a,j) da$$

$$j \in \arg \max_i \int_0^1 [u(c(a,i)) - \phi(a,j)v(y(a,i)) - \eta \mathbf{1}_{[y(a,i)>0]}] da$$

- main results hold for any welfare weights

# Optimal Allocations



## Full information allocation

- Full insurance:  $c(a, j) = c, \forall a, j$

- Consumption-intensive-labor trade-off:

$$u'(c) = \phi(a, j) v'(y(a, j))$$

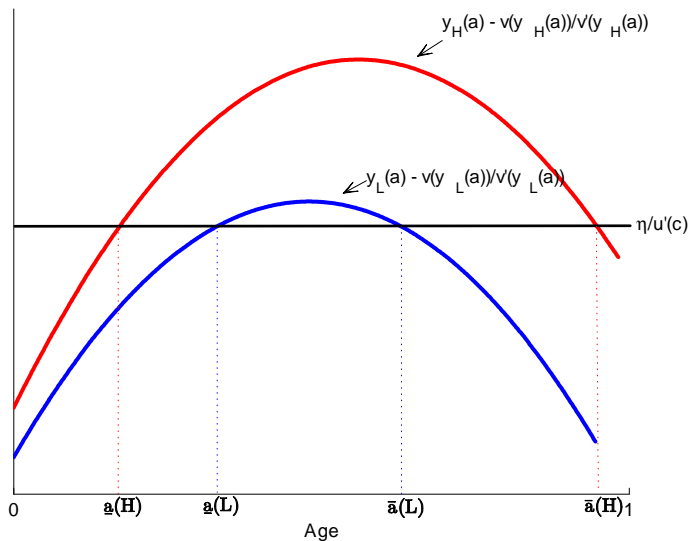
- Consumption-extensive-labor trade-off:  $\forall j \exists \underline{a}_j, \bar{a}_j$  s.t. for  $a \in \{\underline{a}_j, \bar{a}_j\}$

$$u'(c)y(a, j) = \phi(a, j) v(y(a, j)) + \eta$$

- Extensive Margin Equation, for  $a \in \{\underline{a}_j, \bar{a}_j\}$ :

$$y(a, j) - \frac{v(y(a, j))}{v'(y(a, j))} = \frac{\eta}{u'(c)}$$

# Two-type illustration



# Private information allocation - consumption/savings

- $c(a, j) = c(a', j) = c(j)$  for all  $a, a' \in [0, 1]$  except for a measure 0 subset
- No intertemporal shocks  $\Rightarrow$  no saving distortions

# Private information allocation - labor supply

- No distortion at the top along both margins
- Positive consumption-intensive-labor wedge for everyone else

$$u'(c(j)) > \phi(a, j)v'(y(a, j))$$

define

$$1 - \tau_l(a, j) = \frac{\phi(a, j)v'(y(a, j))}{u'(c(j))}$$

# Private information allocation - labor supply

- Positive consumption-extensive-labor wedge for everyone but top type, for  $a \in \{\underline{a}_j, \bar{a}_j\}$

$$u'(c(j))y(a, j) > \phi(a, j)v(y(a, j)) + \eta$$

define

$$1 - \bar{\tau}(j) = \frac{\phi(\bar{a}_j, j)v(y(\bar{a}_j, j)) + \eta}{y(\bar{a}_j, j)u'(c(j))}$$

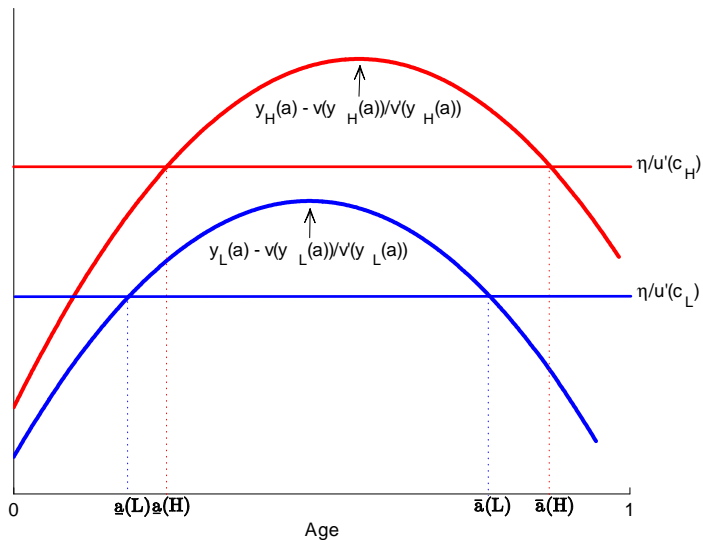
and similarly  $\underline{\tau}$

# Private information allocation - labor supply

- Extensive Margin Equation, for  $a \in \{\underline{a}_j, \bar{a}_j\}$

$$y(a, j) - \frac{v(y(a, j))}{v'(y(a, j))} = \frac{\eta}{u'(c(j))}$$

## Two-type illustration



## Main result - allocation

### Theorem

*In any constrained efficient allocation, independent of Pareto weights and  $\eta$ , wedges satisfy*

$$\bar{\tau}(j) = \tau_l(\bar{a}, j) \frac{1}{\varepsilon(\bar{a}, j)}$$

*where  $\varepsilon(\bar{a}, j) = v'(y(\bar{a}, j)) y(\bar{a}, j) / v(y(\bar{a}, j))$ , and similarly for  $\underline{\tau}$ .*

- Proof: follows from Extensive Margin Equation and the definitions of wedges.
- If  $v(y) = y^{1+1/\gamma} / (1 + 1/\gamma)$ , with constant Frisch elasticity of labor supply  $\gamma$ , then

$$\bar{\tau}(j) = \tau_l(\bar{a}, j) \frac{\gamma}{1 + \gamma}$$



# Implementation

# Implementation

- Try to use policies similar to the ones used in practice
  - nonlinear, progressive taxes on earnings, Social Security or similar pension system
- Implementation depends on market structure
  - we provide an important benchmark of no insurance markets before agents' birth
  - assume equal welfare weights for simplicity

# Main result - implementation

## Theorem

Individual income tax system  $T(y, a)$  implements the optimum iff

- *balanced budget*
- $T_y^-(y^*(a, j), a) < T_y^+(y^*(a, j), a)$   
(smooth tax functions do not implement the optimum, as in Mirrlees)
- $[\phi(\bar{a}_j^*, j) - \phi(\bar{a}_j^*, j + 1)] v(y^*(\bar{a}_j^*, j)) \leq T^+(y^*(\bar{a}_j^*, j), \bar{a}_j^*) - T(y^*(\bar{a}_j^*, j), \bar{a}_j^*)$   
(and analogous condition for  $\underline{a}$  and  $T^-$ )

Interpretation: tax system should provide large disincentives to work after retirement, i.e. jump in taxes

## Conclusion and a way forward

We develop a theoretical framework that ..

- provides a direct test of whether a tax/pension system is Pareto optimal
- qualitatively justifies large disincentives to work after retirement
  - a jump in taxes after retirement age

We are working on ..

- ways of performing the optimality test in the data
- quantitative assessment of U.S. Social Security
- optimal reforms, including in response to demographic shifts